Supplementary Material

Life Disparity

Life disparity, $e^\dagger$, is the life expectancy lost due to death, $e^\dagger = \int_0^\omega e(x,t)f(x,t)dx$, where $e(a,t) = \int_a^\omega \ell(x,t)dx/\ell(a,t)$ is remaining life expectancy at age a and time t, $\ell(a,t) = \exp\left(-\int_0^a \mu(x,t)dx\right)$gives the probability of survival to age a and $\mu(a,t)$ denotes the age-specific hazard of death. The life table distribution of deaths is given by $f(a,t) = \ell(a,t)\mu(a,t)$. Maximum lifespan is denoted by ω.

Conceptually, this measure is similar to Greville's 1948 variant of the Potential Years of Life Lost (PYLL) measure,[S1] which weights the death counts from a given disease at each age by remaining life expectancy in order to assess the importance of major causes of death. In this way the age profile of disease mortality is taken into account, which can lead to different conclusions than assessments that compare diseases strictly on the basis of death counts or on their average effect on lifespan. When all causes of death are taken into account, life disparity functions in much the same way. Saving lives at ages with both many remaining life years and a high number of death counts has the greatest impact on lifespan variation. This was first observed by Keyfitz,[S2,S3] who derived the formula for the elasticity of life expectancy to a proportional change in mortality (also known as the entropy of the life table, or Keyfitz's $\mathcal{H}$), which he observed was related to variation in age-at-death. Life disparity equals the entropy of the life table multiplied by life expectancy.[S4-S6] It is only in recent years that the full potential of life disparity as a measure of lifespan variation has been realized.[S7,S8]

Methods to calculate the contribution of premature and late mortality to changes in life expectancy

Let $\rho(a,t)$ represent the rate of progress in reducing mortality:

$$\rho(a,t) = -\frac{\frac{\partial}{\partial t}\mu(a,t)}{\mu(a,t)}$$

Vaupel and Canudas Romo[S9] showed that the change in life expectancy at birth, $\dot{e}(0,t)$ could be decomposed as:

$$\dot{e}(0,t) = \int_0^\omega e(a,t)\rho(a,t)f(a,t)da$$

Meanwhile, Zhang and Vaupel[S8] proved that this relationship could be further decomposed by age components, for instance premature and late life mortality components separated by the threshold age:

$$\dot{e}(0,t) = \int_0^{a^\dagger(t)} e(a,t)\rho(a,t)f(a,t)da + \int_{a^\dagger(t)}^\omega e(a,t)\rho(a,t)f(a,t)da$$

We used these relationships to calculate the yearly contributions of averting prema-

ture and late deaths to increases in life expectancy, which we then smoothed using a 20-year moving average.

Methods to obtain confidence intervals

To be sure that random fluctuation was not substantially affecting our rankings of life expectancy, life disparity and the threshold age in Table 1, we estimated 95% confidence intervals around our results. This was done by Monte Carlo simulation, assuming a binomial distribution of death counts. For each age interval the number of observations in each simulation round was based on the observed number of deaths, D_x, divided by the probability of dying, q_x. The simulated death counts, d_x^{sim}, divided by the observed population at risk, N_x, gave us simulated death probabilities q_x^{sim}. From these values we simulated 1000 life tables that we used to generate confidence intervals around our life-table-based estimates. Others have used similar methods to generate confidence intervals around life expectancy and healthy life expectancy for small populations.[S10-S13]

The relationship between life expectancy and life disparity: comparing leaders and laggards

In Figure 3 we showed that reaching a level of life expectancy later was only sometimes associated with lower life disparity conditional upon survival to age 15. More often no clear relationship could be found.

Given the high correlation between life expectancy and life disparity, an interesting question is whether laggards in life expectancy at birth also have similar levels of life disparity to the leaders at the same level of life expectancy. The answer, seen in Figure S4, is yes. Thus while leaders are the first to reduce premature mortality, thereby reducing life disparity, laggards on average follow with similar reductions in life disparity alongside life expectancy increases.

Alternative calculations using the AID measure

Some concern might be raised about whether artefactual correlations are present in our findings since calculation of life disparity involves prior knowledge of life expectancy. We showed the high correlation between life disparity and other measures of lifespan variation in Tables S2 and S3 of the supplementary material. Some of these other measures do not contain life expectancy in their formulation. To be sure that our results were robust to other measures, we ran our analysis with an alternative measure of lifespan variation, the absolute inter-individual difference (AID). While AID and life disparity are highly correlated, AID tends to be more sensitive to mortality change at younger ages than life disparity.[S7]

The AID is an alternative measure of lifespan variation that is related to the well-known Gini coefficient of inequality. There are many equivalent formulations to the

Country or region	Earliest year	Latest year
Australia	1921	2007
Austria	1947	2008
Belgium*	1841	2007
Bulgaria	1970	2009
Belarus	1970	2007
Canada	1921	2007
Switzerland	1876	2007
Chile	1992	2005
Czech	1950	2009
West Germany	1956	2008
East Germany	1956	2008
Denmark	1840	2008
Spain	1908	2006
Estonia	1959	2009
Finland	1878	2009
France	1840	2007
England & Wales	1841	2009
North Ireland	1922	2009
Scotland	1855	2009
Hungry	1950	2006
Ireland	1950	2006
Iceland	1840	2008
Israel	1983	2008
Italy	1872	2007
Japan	1947	2009
Latvia	1970	2009
Luxembourg	1960	2007
Lithuania	1959	2009
Netherlands	1850	2008
Norway	1846	2008
New Zealand non-Maori	1901	2008
Poland	1958	2009
Portugal	1940	2009
Russia	1959	2008
Slovakia	1950	2009
Slovenia	1983	2009
Sweden	1840	2008
Taiwan	1970	2009
Ukraine	1970	2006
USA	1933	2007
Australia	1921	2007

Table S1: Countries and regions of the Human Mortality Database used in our analysis. We used data from the earliest year given in the table through the latest year. * No data was available for 1914-1918.

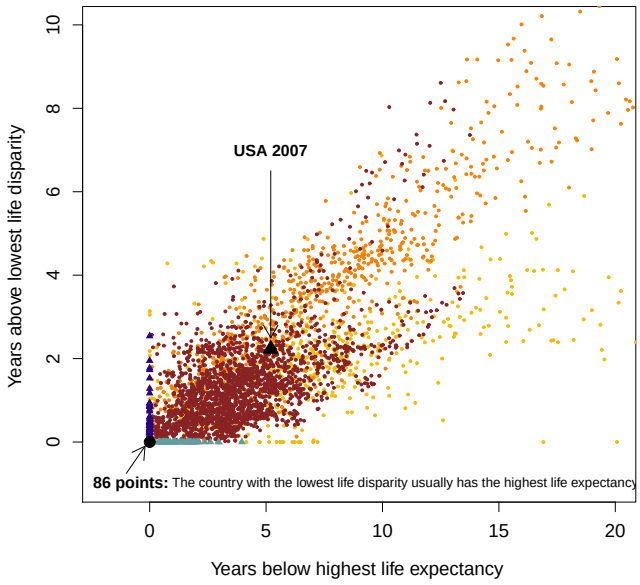

Figure S1: The association between life disparity in a specific year and life expectancy for females in that year for the 40 countries and regions in the Human Mortality Database, 1840-2009 (Table S1). The correlation coefficient between them is 0.75 (95% confidence interval 0.73 to 0.76). The black triangle represents the United States in 2007: the U.S. had a female life expectancy 5.2 years lower than the international record in 2006 and a life disparity 2.2 years greater. The brown points denote years after 1950, the orange points 1900-1949 and the yellow points 1840-1900. The light blue triangles represent countries with the lowest life disparity but with a life expectancy below the international record in the specific year; the dark blue triangles indicate the life expectancy leaders in a given year, with life disparities greater than the most egalitarian country in that year. The black point at (0,0) marks countries with the lowest life disparity and the highest life expectancy. During the 170 years from 1840 to 2009, 86 holders of record life expectancy also enjoyed the lowest life disparity.

	$e^\dagger$	σ^2	σ	σ_{10}	$\mathcal{H}$	G	IQR	AID
Life disparity ($e^\dagger$)	1.000							
Variance (σ^2)	0.993	1.000						
Standard deviation (σ)	0.985	0.996	1.000					
Standard deviation past age 10 (σ_{10})	0.972	0.964	0.961	1.000				
Entropy of life table ($\mathcal{H} = e^\dagger/e(0)$)	0.966	0.936	0.919	0.916	1.000			
Gini coefficient (G)	0.983	0.961	0.946	0.937	0.997	1.000		
Inter-Quartile Range (IQR)	0.967	0.944	0.917	0.921	0.966	0.974	1.000	
Inter-individual difference (AID)	0.995	0.998	0.996	0.973	0.937	0.962	0.945	1.000

Table S2: Pearson correlation coefficients between pairs of measures of lifespan variation, based on all 3528 female period life tables available from the Human Mortality Database, 1840-2009.

	$e^\dagger$	σ^2	σ	σ_{10}	$\mathcal{H}$	G	IQR	AID
Life disparity ($e^\dagger$)	1.000							
Variance (σ^2)	0.986	1.000						
Standard deviation (σ)	0.979	0.996	1.000					
Standard deviation past age 10 (σ_{10})	0.940	0.909	0.908	1.000				
Entropy of life table ($\mathcal{H} = e^\dagger/e(0)$)	0.958	0.913	0.898	0.879	1.000			
Gini coefficient (G)	0.979	0.946	0.933	0.898	0.996	1.000		
Inter-Quartile Range (IQR)	0.965	0.937	0.913	0.890	0.948	0.964	1.000	
Inter-individual difference (AID)	0.992	0.997	0.995	0.930	0.917	0.950	0.941	1.000

Table S3: Pearson correlation coefficients between pairs of measures of lifespan variation, based on all 3528 male period life tables available from the Human Mortality Database, 1840-2009.

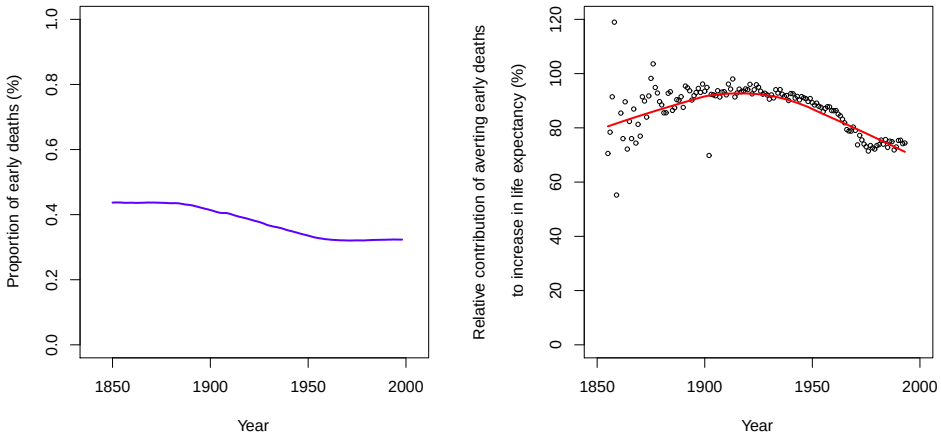

Figure S2: The left panel expresses early deaths as a proportion of all deaths, smoothed by 20-year averages. The right panel displays the 30-year moving average of the relative contribution of averting early deaths to the increase in life expectancy, with the red line marking the trend. The data pertain to females, 1840-2009, all 40 countries and regions of the HMD.

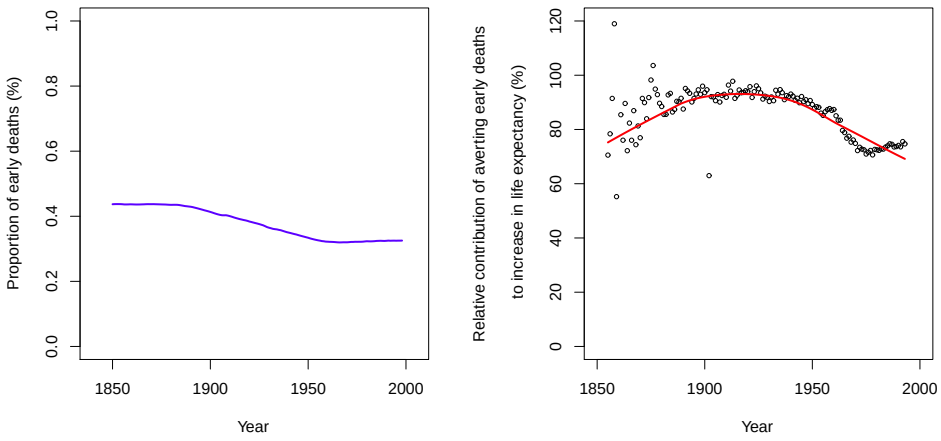

Figure S3: To show that the trends in Figure S2 above are not due to compositional change from new entrants into our dataset, we plotted the two relationships using only the eleven countries for which we had over 100 years of data (see Table S1).

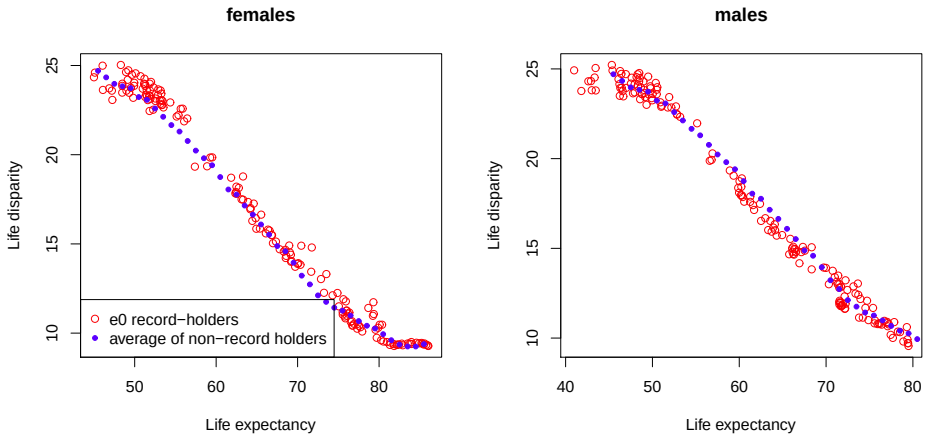

Figure S4: A comparison of the level of life disparity between (1) record-holders of life expectancy and (2) all other countries (averaged) once they had reached certain life expectancy levels (taken to be one year intervals).

AID, but the Kendall and Stuart definition is the most helpful for understanding the nature of the statistic, which essentially measures the average absolute distance in years between each pair of individuals' age at death (length of life) in the population.[S14] From the life table, it can be calculated as follows:

$$AID = \frac{1}{2}\int_0^{\omega}\int_0^{\omega} |x-y| f(x) f(y) dx dy$$

where $|x-y|$ is the absolute value of the distance in years between age x and age y, and $f(x)$ and $f(y)$ are the probabilities of death at ages x and y respectively.

Using the AID measure, the country with the highest life expectancy also had the lowest AID 74 times for females (Figure S5), and 67 times for males (Figure S6) out of 170 years. Differences in this relationship between the two measures were mostly owing to differences in historical populations, especially during war, famine and epidemic years, when certain countries had qualitatively different age at death distributions from other countries.

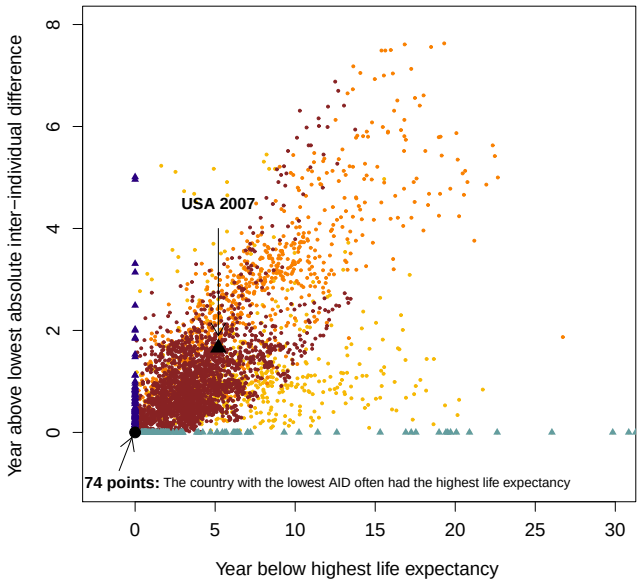

Figure S5: Alternative calculations using the AID measure, females. The correlation coefficient between them is 0.60 (0.58 to 0.62). The brown points denote years after 1950, the orange points 1900-1949 and the yellow points 1840-1900. The light blue triangles represent countries with the lowest life disparity but with a life expectancy below the international record in the specific year; the dark blue triangles indicate the life expectancy leaders in a given year, with life disparities greater than the most egalitarian country in that year.

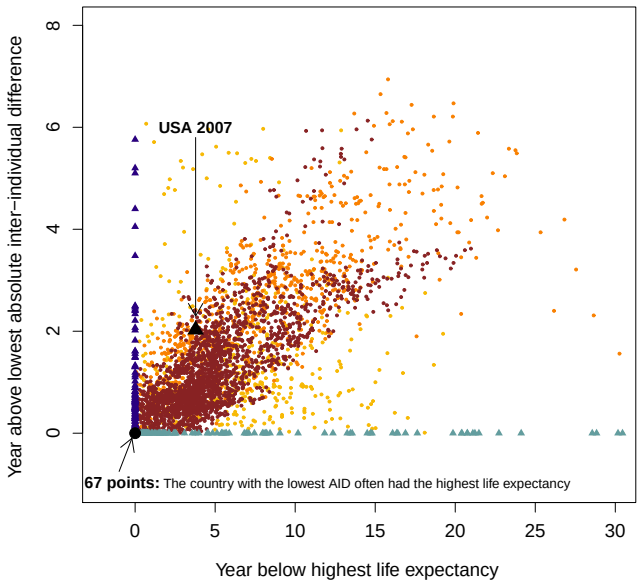

Figure S6: Alternative calculations using the AID measure, males. The correlation coefficient between them is 0.62 (0.60 to 0.64). The brown points denote years after 1950, the orange points 1900-1949 and the yellow points 1840-1900. The light blue triangles represent countries with the lowest life disparity but with a life expectancy below the international record in the specific year; the dark blue triangles indicate the life expectancy leaders in a given year, with life disparities greater than the most egalitarian country in that year.

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