

Methodological appendix: The age-period-cohort-interaction model

Many methods have been developed under the traditional age-period-cohort (APC) framework including the intrinsic estimator and the hierarchical APC model. In addition to methodological concerns,^{1–3} these methods assume that cohort effects can be differentiated when the influence of social changes is uniform across age groups. That is, regardless of the identification strategies, they all attempt to identify the independent and additive effects of age, period and cohort on crime trends. However, it has been argued that this traditional framework contradicts the demographic literature that clearly defines cohort effects in the age-period specification.^{4–7} The age-period-cohort-interaction (APC-I) model conceptually differs from the traditional APC by explicitly modeling cohort effects as an interaction between age and period main effects; that is, considering cohort effects as the differential effects of social changes and events depending on age. By so doing, the APC-I model is consistent with the conceptualization of cohort effects. It acknowledges the inherent interactive nature of age, period and cohort and does not attempt to estimate the linear or nonlinear cohort effects that are independent or additive to age and period effects as defined in the problematic traditional APC framework.

In its simplest form, the APC-I model can be written as:

$$g(E(Y_{ij})) = \mu + \alpha_i + \beta_j + \alpha\beta_{ij(k)}, \quad (1)$$

where $E(Y_{ij})$ denotes the expected obesity status Y for the i th age group in the j th period of time; g is the "link function" (e.g., logit for a binary outcome); α_i denotes the mean difference from the global mean μ associated with the i th age category; β_j denotes the mean difference from μ associated with the j th period; $\alpha\beta_{ij(k)}$ denotes the interaction of the i th age group and j th period group corresponding to the effect of the k th cohort. Note that except for the youngest and

oldest cohorts, the effect of one cohort includes multiple age-by-period interaction terms $\alpha\beta_{ij(k)}$ as the cohort ages over time. To investigate regional variation in temporal patterns in obesity for each race-sex group, we extend the APC-I model to include a three-way interaction term among region, age and period and estimate the model separately by race-sex groups. Specifically, we estimate the following model for each race-sex group with and without socioeconomic covariates:

$$g(E(Y_{ij})) = \mu + \alpha_i + \beta_j + region_l + \alpha\beta_{ij(k)} + region_l \times \alpha_i + region_l \times \beta_j + region_l \times \alpha\beta_{ij(k)}, \quad (2)$$

where $E(Y_{ij})$, g , μ , α_i , β_j , and $\alpha\beta_{ij(k)}$ are defined earlier; $region_l$ denotes the main effects of residing in region l ; the interaction terms between $region_l$ and α_i , β_j , and $\alpha\beta_{ij(k)}$ represent regional heterogeneity in age, period, and cohort patterns.

For each demographic group, we began with an APC-I model with all two-way and three-way interactions. Results of the full model suggest little regional variation in age or period trends ($p>.3$ for all demographic groups) but significant cohort effects (characterized as age-by-period interactions, $p<.001$ except for White men) and substantial regional variations among cohorts for Black men and White women ($p<.001$). We thus focus on the more parsimonious model without the two-way interaction between region and age or period in subsequent analysis.

A distinct type of APC models that is also theoretically informed is the mechanism-based model.⁸ The mechanism-based model requires researchers to specify age-, period-, and cohort-specific mechanisms (e.g., education or relative cohort size as a cohort-related mechanism) under a causal inference framework to identify the unique contribution of the three variables to homicide trends. This causal approach is beyond the scope of the current research, but we

consider our APC-I analysis as a first step towards understanding the mechanisms underlying age, period, and cohort effects on obesity.

Reference

1. Luo L. Assessing Validity and Application Scope of the Intrinsic Estimator Approach to the Age-Period-Cohort Problem. *Demography*. 2013 Dec 1;50(6):1945–67.
2. Luo L, Hodges JS. Constraints in Random Effects Age-Period-Cohort Models. *Sociological Methodology*. 2020 Aug 1;50(1):276–317.
3. Bell AJ, Jones K. Another “futile quest”? A simulation study of Yang and Land’s Hierarchical Age-Period-Cohort model. *Demographic Research*. 2014 Feb 4;30(10.4054/DemRes.2014.30.11):333–60.
4. Ryder NB. The Cohort as a Concept in the Study of Social Change. *American Sociological Review*. 1965;30(6):843–61.
5. Hobcraft J, Menken J, Preston S. Age, Period, and Cohort Effects in Demography: A Review. *Population Index*. 1982;48(1):4–43.
6. Luo L. Paradigm Shift in Age-Period-Cohort Analysis: A Response to Yang and Land, O’Brien, Held and Riebler, and Fienberg. *Demography*. 2013 Dec 1;50(6):1985–8.
7. Luo L, Hodges JS. The Age-Period-Cohort-Interaction Model for Describing and Investigating Inter-cohort Deviations and Intra-cohort Life-course Dynamics: *Sociological Methods & Research* [Internet]. 2020 Jan 23 [cited 2020 Oct 8]; Available from: <https://journals.sagepub.com/doi/10.1177/0049124119882451>
8. Winship C, Harding DJ. A Mechanism-Based Approach to the Identification of Age–Period–Cohort Models. *Sociological Methods & Research*. 2008 Feb 1;36(3):362–401.